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Franco-Scottish Science Seminar
Linear Algebra and Parallel Computing at the Heart of Scientific Computing
Wednesday 21 September 2016

The Franco-Scottish Science Seminar series is co-sponsored by the RSE and the French Embassy in London in the context of a programme of science events designed to explore and publicly present areas of science where both Scotland and France have a powerful presence. The topic chosen for this year's seminar was "Linear Algebra and Parallel Computing at the Heart of Scientific Computing".

Most numerical computations and simulations have at their core the solution of systems of linear equations. As the computations increase in scale and the simulations become more detailed, the size of such systems increases. The seminar was focused on recent research into improving the solution of such large systems and some novel applications that can now be addressed. Most of this research has involved direct collaboration between Scotland and France. In fact, two main axes of the presentations concerned joint projects involving researchers from France and Scotland: one on domain decomposition methods centred in Paris and the other on intelligent matrix partitioning involving researchers in Strathclyde, Toulouse, and Lyon.

The topics covered by the six invited lectures were designed to appeal to researchers in numerical methods and scientists and engineers involved in numerical simulation related to innovation in aerospace, semiconductor design, the chemical an oil industry, transport, energy, telecommunications, medical imaging and diagnosis, non-destructive testing and financial mathematics.

The first talk was presented by a CNRS Senior Scientist from the Jacques Louis Lions Laboratory at the University of Pierre and Marie Curie in Paris. Frédéric Nataf spoke on "A theory for the P.L. Lions Algorithm in the Framework of Optimized Schwarz Methods". Optimized Schwarz methods (OSM) are very popular methods which were introduced by P.L. Lions for elliptic problems and by B. Despres for propagative wave phenomena. One drawback is the lack of theoretical results for variable coefficient

problems and overlapping decompositions. In this talk, Nataf discussed building a coarse space for which the convergence rate of the two-level method is guaranteed regardless of the regularity of the coefficients. He showed this by introducing a symmetrized variant of the ORAS (Optimized Restricted Additive Schwarz) algorithm by A. St Cyr et al. and by identifying the problematic modes by using two different generalized eigenvalue problems instead of only one as for the ASM (Additive Schwarz Method), BDD (Balancing Domain Decomposition) or FETI (Finite Element Tearing and Interconnection) methods. The algorithms are implemented in the HPDDM library interfaced with the multi-physics finite-element software package FreeFem++. The software is open source under LGPL licences and makes it possible to perform scalable very large scale simulations.

The first "Scottish talk" was by Philip Knight from the University of Strathclyde who spoke on "Uncovering Hidden Block Structure". He first applied a two-sided diagonal scaling to a nonnegative matrix to render it into doubly stochastic form. This is possible if and only if the matrix is fully indecomposable. The scaling often reveals key structural properties of the matrix as the effects of element size and connectivity are balanced. By exploiting key spectral properties of doubly stochastic matrices, it can be shown how to use the scaling to reveal hidden block structure in matrices without any prior knowledge.

The final talk of the morning session was given by Daniel Ruiz from ENSEEIHT-IRIT in Toulouse who continued the theme of the previous talk by a presentation on "Identifying Blocks in Matrices with Information from both Numerical Values and Sparsity Pattern". He expanded on the talk of Knight and emphasized more the mathematical underpinning of the algorithms. He used the spectral property of doubly stochastic matrices to reveal the block structure by detecting steps in the eigen functions. This was combined with classical graph analysis techniques to design partitioning algorithms for large sparse matrices, based on both numerical values and pattern information.

The first afternoon talk was by Victorita Dolean of the University of Strathclyde who spoke on "Microwave Tomographic Imaging of Cerebrovascular Accidents by Using High-Performance Computing". Her research team did this work in collaboration with EMTensor, an Austrian innovative SME, dedicated to biomedical imaging. For the first time in the world, they had demonstrated on synthetic data the feasibility of a new imaging technique based on microwaves, allowing both the characterization of the CVA beginning with the patientcare in an ambulance and then through continuous monitoring of the patient during hospitalization. In order to develop a robust and precise methodology for microwave imaging, a few distinct research fields needed to be mastered: optimization, inverse problems, approximation and solution methods for the simulation of the direct problem modelled by Maxwell's equations.

The precise simulation of a direct problem for a complex and highly heterogeneous medium is a challenge in itself. A few tools already developed by the researchers of the team were used: the HPDDM library for domain decomposition and its interface with the FreeFem++ software (finite elements). The medical and industrial challenge of this work is very important. It is the first time that such a realistic study demonstrates the feasibility of microwave imaging. Although it is less precise than RMI or CT scan, its low price, its reduced size and its lack of harm even in a continuous use could make microwave imaging of the brain the equivalent of what echography (ultrasound imaging) brings to the exploration of other parts of the human body.

The final "French" talk was given by Bora Uçar, a CNRS Scientist from ENS-Lyon. He spoke on "Bipartite Matchings in Solving Sparse Linear Systems". He covered the use of weighted matching algorithms for bipartite graphs in solving sparse linear systems with both direct and with iterative methods. For direct methods, he showed how the matchings can be used to facilitate pivoting in order to reduce both time and storage requirements. For iterative methods, he showed how such matchings can be used to generate effective preconditioners.

The closing talk by Peter Richtárik from the School of Mathematics and the University of Edinburgh was entitled "Randomized Iterative Methods for Solving Linear Systems". He developed a parametric family of reformulations of an arbitrary consistent linear system into a stochastic problem. His reformulation had several equivalent interpretations, allowing researchers from various communities to leverage their domain specific insights. In particular, his stochastic reformulation can be equivalently seen as a stochastic optimization problem, a stochastically preconditioned linear system, a stochastic fixed point problem and as a probabilistic intersection problem. He proved sufficient, and necessary and sufficient conditions for the reformulation to be exact. He then proposed and analysed several stochastic algorithms for solving the reformulated problem, with linear convergence rates. The complexity of his basic method is O (kappa), where kappa is a condition number associated with the stochastic reformulation, and can be adjusted by choosing an appropriate parameter defining the reformulation. The complexity of his accelerated method is O (sqrt(kappa)). He also presented a parallel method, whose complexity improves with the level of parallelism. His methods can be interpreted as basic and accelerated variants of stochastic gradient descent, stochastic Newton descent, stochastic projection method and a stochastic fixed point method, with fixed stepsize (relaxation parameter), applied to the reformulation. His framework (reformulation + algorithms) includes several known randomized methods for solving linear systems, such as the randomized Kaczmarz and block Kaczmarz methods. However, his framework is substantially more general, and his analysis is deeper. He not only recovers known results in these special cases, but also improves upon them, and gains new insights.

In conclusion, a most enjoyable and constructive day was had by all participants aided by the smooth organization of the event by the RSE and the French Embassy in London.