The Royal Society of Edinburgh

Inspiring Brilliance:
Celebrating Maxwell's Genius and Legacy

on the 150th Anniversary of the Publication of Maxwell's Equations

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Report by Peter Barr

James Clerk Maxwell: The story continues

"Inspiring Brilliance" brought together some of the most distinguished scientists in the UK to celebrate the genius and the legacy of one of the world's greatest physicists, considered the equal of Newton and Einstein, whose achievements are only beginning to get the public recognition they deserve...

It's almost unbelievable that one man could have such an impact on so many different areas of science and technology: digital communications, colour photography and cybernetics; mathematics, structural mechanics and astronomy; statistical physics and particle physics; relativity and quantum theory. And his influence not only continues 150 years after he published his ground-breaking theories, but promises to carry on into the future.

The man who "did it all" was a scientist from Scotland called James Clerk Maxwell, born in Edinburgh in 1831. And he first came to public attention when James Forbes, the Professor of Natural Philosophy at the University of Edinburgh, presented the young teenager's paper *On the description of Oval Curves and those having a plurality of Foci* at the Royal Society of Edinburgh (RSE) in 1845.

At the RSE 170 years later, an audience was treated to a day of presentations which reviewed the key achievements of Maxwell's career and his continuing impact on science and technology, as well as focusing on some of his less famous achievements – for example, he devised a foolproof method of measuring the precise length of a yard, and he wrote poetry in his spare time.

RSE President Professor Dame Jocelyn Bell Burnell welcomed the audience and not only drew attention to the 43 poems by Maxwell, but also cleared up all confusion regarding his name – he had been born James Clerk but gained the name Maxwell when his father added it in order to inherit a family estate. So, strictly speaking, Maxwell should be known as Clerk Maxwell.

The Chairman of the James Clerk Maxwell Foundation, David Forfar, introduced the presentations on Maxwell by quoting the Nobel-prize-winning physicist Richard Feynman: *"Ten thousand years from now, there can be little doubt that the most significant event of the 19th Century will be judged as Maxwell's discovery of the laws of electrodynamics. The*

American Civil War will pale into provincial insignificance in comparison with this important scientific event of the same decade."

Forfar also described Maxwell's close connections with the RSE, beginning with his paper of 1845 (at age 14) and two subsequent papers, before he even entered University. He became a Fellow of the RSE at the age of 24 and twelve other family members were Fellows of the RSE or of the Royal Society, or both. And when Maxwell was interviewed at Trinity College in Cambridge in 1850, he was able to quote these three RSE papers as examples of his work.

All the speakers focused on Maxwell's huge impact, spanning three centuries – and promising much more to come. Emeritus Professor Peter Higgs revealed that the work of the Large Hadron Collider at CERN may be on the verge of some earth-shattering discoveries, including dark matter, with Maxwell's theories still a significant factor in the latest research.

Professor Harald Haas did a world's-first demonstration, transmitting large amounts of encoded video data at very high speed via red, green and blue LED lights, using "a modern version of Maxwell's light box which will soon be inside every smartphone."

In other words, new chapters in the Clerk Maxwell story continue to open; not just in fundamental science, but in everyday technology.

1. Maxwell's Equations: The Tip of an Iceberg

Professor Peter Higgs CH FRS FRSE – Nobel Laureate; Emeritus Professor of Theoretical Physics, University of Edinburgh

"I'll try to tell the story of Maxwell's equations, and how they are only the tip of an iceberg," Professor Higgs began, before he traced the influence of Maxwell on the history of physics. He described various attempts to progress towards a unified theory of all forces – a "theory of everything" – in the midst of which Maxwell's equations continue to play a key role.

Maxwell's theory of electromagnetic fields was the "first fully-fledged field theory of forces" said Higgs. Faraday had talked about electric and magnetic fields before, but Maxwell advanced this and came up with the first classical field theory of electromagnetism. Maxwell was still basically "Newtonian" and lived at a time when many scientists still believed in the "luminiferous aether" – the idea of a medium through which light could travel – and many believed that light was made of subtle particles. Maxwell then came up with his theory that light was composed of electromagnetic waves. The failure of experiments to find the aether was used by Einstein to develop his special theory of relativity in 1905. Forty years after Maxwell's electromagnetism, Einstein resurrected the particle, or corpuscular, theory of light and modified Newton's ideas, leaving Maxwell's theories intact. When Einstein developed his theory of general relativity in 1915, he did so "with the knowledge of what went before him," including Maxwell's work on electromagnetism.

Quantum mechanics and the development of quantum electrodynamics owe a large debt to Maxwell's equations, said Higgs, and Maxwell's influence on physics continued throughout the 20th Century. Higgs discussed the "problems of infinities" which accompanied the emergence of quantum field theory, which incorporated generalisations of Maxwell's equations. The next development, said Higgs, was the connection between two major concepts in physics – symmetries and conservation. And as physicists wrestled with various theories over the decades, asking "what to do with symmetry?" and describing complex topics such as wave equations of charged particles in electromagnetic fields, Maxwell's theories always had a central role to play.

"During the course of the 20th Century," Higgs continued, "it became clear that the forces of electromagnetism and gravity were not the whole story." Theories were developed to take account of nuclear beta decay and radioactivity, and new ideas emerged describing phenomena such as the "weak nuclear force." Rutherford also did important work on the atomic nucleus, while other new theories suggested that there must be a "strong force" to hold the nucleus together. In 1932, ideas such as "isotopic spin" appeared. "The story of elementary particles involves the discovery of bigger and bigger apparent symmetries, associated with 'charge' conservation," said Higgs. But the theory of strong nuclear force "didn't work too well" when you tried to understand neutrons and protons. And the theory of weak force was "even worse." It was clear there were similarities between weak force and electromagnetic force, but if there was, said Higgs, "the particles that carry the weak force would have to be massive." When you try to handle the quantum theory involving massive force-carriers, he continued, you get "nonsense" and it is a "disaster for the theory."

In the midst of these struggles, it was "easy to generalise Maxwell's equations to higher symmetries." Fields now interacted with themselves, but massless particles were "another non-starter," said Higgs. Then in 1960, quantum theory "smashed the theory" of superconductivity and the spontaneous breaking of symmetry – physicists assumed that a vacuum would be symmetric, said Higgs, but this was not the case. The appearance of "massless, spin-less particles was a disaster," he said, "because there aren't any."

Fifty years ago, said Higgs, physicists found a way out of this "difficulty," and Maxwell-type fields helped them develop new theories. All that had to be done was "marry spontaneous symmetry with Maxwell field theory," to produce a theory that was later proved to be experimentally correct, followed by other "viable" theories such as quantum chromodynamics, in which "nothing was swept under the carpet," he added.

We now know that "neutrons and protons are not elementary particles," said Higgs, "but are made of quarks." And Maxwell-type fields have continued to play a big role in the development of particle physics.

"Maxwell has spawned a lot of progeny," Higgs concluded.

Q: When can we expect the unification of general relativity theory? Will it be when "Maxwell marries Einstein"?
A: There is a class of theories that can't handle the inherent infinities. In the 1970s, there was a promising theory of super-symmetry, predicting that the elementary particles (of the Standard Model) would each have a super-symmetric partner. This idea could solve some of the problems of uniting the field theories of some of the forces. Super-symmetry in turn triggered the development of string theory and super-string theory, avoiding some of these difficulties in quantum field theory, but the timescale for a fully successfully unification of all forces (including gravity) is another question.

Q: What will we discover when CERN (the Large Hadron Collider) is boosted?
A: CERN is now running at close to the design energy (of 14 TeV). The discoveries three years ago were at half the design energy. It maybe that types of super-symmetric particles will be found, including candidate particles for dark matter – which makes up 90 per cent of the Universe. There may of course be some surprises – things we did not expect. There will also be more detailed measurements, to help us understand things found three years ago.

Q: How heavy will these particles be?
A: Theories predict some of these super-symmetric particles will be much lighter than others. The masses of some of these may be within reach of the present CERN machine, at least in the simpler versions of super-symmetry theory.

2. Maxwell, Field Theory and the Road to Relativity and Quantum Theory

Sir Peter Knight FRS – Senior Research Investigator at Imperial College London; Senior Fellow in Residence at Chicheley Hall and Past President of the Institute of Physics

”Newton hedged his bets on the theory of light,” Professor Knight began. The corpuscular view had been the dominant theory for almost 200 years, but Maxwell made the breakthrough in 1865, when he worked out that “the speed of propagation of an electromagnetic field is approximately that of the speed of light.“ Faraday and Maxwell were the first to introduce the theory of fields, and others later extended these ideas, but Maxwell was the one who made the breakthrough.

As well as coming up with his brilliant equations, Maxwell built “ingenious models” to demonstrate his theories, and introduced the concept of the “displacement current,” which he incorporated into his landmark paper *A Dynamical Theory of the Electromagnetic Field*.

Knight then traced the history of Maxwell's great paper, and displayed an image of the handwritten manuscript that was sent for peer review to William Thompson (Lord Kelvin) and George Stokes at the Royal Society, showing that there was some ambiguity about the submission date on the paper – the year 1863 was scored out and a four was inserted replacing the three. The paper was subsequently presented at the Royal Society in December 1864 and published in November the following year. Kelvin's referee report stated that he had “read it nearly through with great interest.”

Maxwell's important conclusion was that, “Light and magnetism are affections of the same substance.” And Knight said that his theory of electromagnetism was the first success in unification theory, bringing together electricity, magnetism and light. Rutherford extended Hertz’s experimental work to verify Maxwell's theory’s prediction of electromagnetic waves for the first transmission over reasonable distances, and Marconi then applied it in developing radio technology, so that electromagnetic waves now “whizz around the world,” said Knight. “Light powers the Internet world.” Maxwell's work was also a relativistic theory which was compatible with the special theory of relativity; “the generally accepted and experimentally well confirmed physical theory regarding the relationship between space and time.”

Maxwell was also “the precursor” of the principles of relativity when he developed his theory of invariant light speed. In 1860, Maxwell demonstrated the relationship between electricity and magnetism and, over a century later, scientists made a similar breakthrough when they showed the relationship between electromagnetism and weak force, leading to theories of super-unification. Knight also discussed how the physicist Paul Dirac quantified Maxwell's theory, and showed how Maxwell-type fields had a place in what became the theories of quantum mechanics and quantum electrodynamics.

Knight also described one of Maxwell's less famous achievements – the establishment of the SI (International System of Units) unit of measurement for an imperial yard. This had been established in 1824 using a brass rod with gold studs embedded in the House of Commons in London, but after it was destroyed in the burning of Parliament in 1834, Maxwell suggested that the measurement be based on the “imperishable” nature of molecules, the seed of the basic idea of the SI system. .

Theories of light and electromagnetism have transformed our world, Knight concluded, and Maxwell made a huge contribution.

Q: Did Maxwell have mathematical intuition?
A: Faraday had experimental intuition, sometimes described as dreamlike. He also had an allergy to maths. Maxwell had a rigorous training in maths in Edinburgh and would quantify things, but he also built mechanical models – then "stripped out the scaffolding" to see if the theory still worked. Faraday liked to visualise things intuitively, and even though Maxwell liked to do the same, he would always do detailed and sophisticated mathematical calculations.

3. Clerk Maxwell's Influence on Mathematics

Sir Michael Atiyah OM FRS HonFRSE HonFREng – Past President of the Royal Society and the Royal Society of Edinburgh

Maxwell "sailed across many oceans", Sir Michael began, explaining that he himself was "a latecomer to physics", unlike his near contemporary, Emeritus Professor Peter Higgs. Sir Michael then defined Maxwell's place in the history of mathematics and physics, saying that Maxwell was the man who explained light. Sir Michael then mapped out the history of mathematics and physics, starting with Euclid and geometry, moving on from directions to dimensions, and the "templates of chemistry" to equations and algebra, which underpin physics.

Sir Michael continued to describe the different aspects of mathematics, discussing heat equations and wave equations, and how it took so many centuries to work out the square root of minus one – which now lies at the foundation of physics. He also admitted that he still "struggled to understand spinors". "But nobody has really understood spinors", he added.

Next, Sir Michael talked about the non-relativistic world and second-order equations, which help to measure distances, Laplace equations and the mass associated with energy levels In the relativistic world, he continued, we have conformal geometry, "the square root of geometry", rather than classical geometry.

The Archimedes Screw was used by Sir Michael to illustrate various points, particularly gravitational forces. "You can't escape gravity" he said. "It's always there and affects everything, even in nuclear physics." The Archimedes Screw used a double helix design to enable water to "go uphill" and water the crops, and Sir Michael suggested that this gravity-defying method may be how people managed to water the Hanging Gardens of Babylon – "a brilliant ecological solution."

"There is perfect symmetry at the beginning," Sir Michael continued, "but over time, we get asymmetry." Finally, reviewing the evolution of science, he said there were many "past prophets," including Maxwell and he ended with the remark by A.N. Whitehead: "It is more important that a proposition be fruitful than it be true."

Q: What mathematical structures did Maxwell invent – and how did he do it?
A: Maths creates itself, and the same idea permeates all physics and mathematics.

4. The Impact of Maxwell's Work on Colour and Statistical Physics

Professor Malcolm Longair CBE FRS FRSE – Jacksonian Professor Emeritus of Natural Philosophy and Director of Development of the Cavendish Laboratory, University of Cambridge

Maxwell wrote on many different topics, Professor Longair began, including mathematics, electromagnetism, instruments, Saturn's Rings, optics and so on. His presentation focused on Maxwell's contributions to the theory of colour and his work on gases and statistical physics.

As a child, Maxwell was fascinated by the latest optical toys, including Phenakistoscopes and Zoetropes, which work because of the persistence of vision – the human eye can only detect about 20 images per second. Maxwell later improved the design of the Zoetrope, using concave lenses to focus the image on the axis of the drum so that the images looked more realistic. He also used the Zoetrope to demonstrate various aspects of physics such as "stable interacting frictionless vortices" as a model for molecules.

One of Maxwell's early achievements was his quantitative analysis of the three-colour theory of light, which explained the cause of colour blindness if one of the three-colour receptors on the retina was missing. Maxwell demonstrated this first by his colour wheel and then, more precisely, by his light box. In this ingenious device, sunlight is split into a spectrum of precisely controllable amounts of red, green and blue light, which can then be recombined to form white light or any other colour. He determined precisely how much of each of the primary lights is needed to create any colour and produced the first three-colour chromaticity diagram. Maxwell later used his understanding of these principles to take "the world's first colour photograph," a topic covered in more detail in another presentation.

Longair then focused on Maxwell's contribution to the dynamical theory of gases and the two laws of thermodynamics, building on the work of Rudolf Clausius and William Thomson (Lord Kelvin). Maxwell solved the problem of the distribution of speeds in a gas, now known as the Maxwell distribution and, in the process, introduced statistical concepts into the kinetic theory of gases and the laws of thermodynamics. Longair illustrated Maxwell's work on "statistics and thermal distributions" with a number of computer simulations, which showed how random motions lead naturally to the Maxwell distribution and to the second law of thermodynamics, according to which entropy increases spontaneously with time. The model also shows that in statistical equilibrium, there must necessarily be fluctuations all the time, the energies of individual atoms and molecules changing from moment to moment. Maxwell's fundamental contributions were subsequently used by Ludwig Boltzmann and Josiah Willard Gibbs to create the foundations of the modern theory of statistical mechanics.

Maxwell carried out experiments to test the validity of the kinetic theory of gases by studying how the viscosity of gases depends on temperature and pressure. Although the pressure dependence agreed with the predictions of theory, the temperature dependence was wrong. To solve the problem, he replaced the particles by point sources with a repulsive field of force varying as $1/r^5$. Particle collisions were replaced by the interaction between repulsive fields. This was a further example of the move away from the mechanical view of the Universe to one dominated by the interaction between fields.

Longair then discussed Maxwell's thinking on the kinetic theory of gases and 'Maxwell's Demon' (also discussed in more detail in a later presentation), which demonstrates how the second law of thermodynamics has only a "statistical certainty" – the second law is violated all the time at the molecular level but, by considering the statistics of vast numbers of particles, it is overwhelmingly likely that energy will flow from a hotter to a colder body.

Finally, Longair described Maxwell as a “romantic hero” who made possible physics as we know it today.

Q: On a more mundane level, what was the name of Maxwell's dog (incorporated at the suggestion of Longair into the statue of Maxwell erected by the RSE in 2008)?
A: All Maxwell's dogs were called Toby. They accompanied him on his daily visits to his students in the laboratory

Q: How close was Maxwell to discovering quanta?
A: The credit for introducing energy elements into statistical mechanics goes to Boltzmann who knew Maxwell’s work backwards. They are necessary to make the statistical treatment possible. Planck used these ideas in his famous paper on quanta of 1900, showing that energy elements could not become vanishingly small in order to explain the form of the black-body radiation spectrum.

5. The Impact on Astronomy of Maxwell’s Adams Prize Essay
Professor Carl Murray – Professor of Mathematics and Astronomy, Queen Mary University of London

Professor Murray first came across James Clerk Maxwell in a book he read in the early 1970s which described Maxwell's life and work. A page of figures from Maxwell’s essay *On the Stability of the Motion of Saturn's Rings* looked like “hieroglyphics” at the time, but Murray was impressed by the diagram of a model constructed by Maxwell, a mechanical device with wheels and ivory balls designed to demonstrate the motion of the rings around Saturn.

Maxwell's paper won the Adams Prize in 1859. Before then, Saturn's rings had been a mystery, but most astronomers believed that they were fluid, rather than solid. Maxwell's paper disproved both these theories and by analysing the radial, tangential and normal components of a displaced satellite in a circular ring showed that stable waves could be set up. He also studied how one such ring would affect another, before concluding that the rings of Saturn are “composed of an indefinite number of unconnected particles, revolving around the planet with different velocities according to their respective distance.”

Maxwell designed his Saturn machine to demonstrate the motions of the rings “for the edification of sensible image worshippers,” acknowledging the need for visual aids to understanding. The model, now at the Cavendish Laboratory, was built in Aberdeen in 1858 and shows a wave pattern which is set up in a ring of interacting satellites.

The mathematician and astronomer Sir George Biddell Airy wrote that Maxwell’s essay was “one of the most remarkable contributions to mechanical astronomy,” but Murray said “its immediate impact was “practically nil.” A major work on Saturn’s rings published in 1872 did not even mention the paper by Maxwell, but by the time James Edward Keeler published his work on spectroscopic observations of the rings in 1895, Maxwell was duly cited and his contribution had been firmly established. More than a century later, images of the rings from the Cassini spacecraft regularly confirm Maxwell's conclusion. The rings consist of particles, mostly composed of water ice with some impurities, but their origin is still uncertain although probably due to the break-up of a large comet or moon. Maxwell did overestimate the thickness, however – not 100 miles but only ten metres.

Many waves have been detected in Saturn’s rings – some are set up spontaneously and some are forced by the orbiting satellites. The edge of the main rings has been confirmed to have a seven-lobed figure reminiscent of that studied by Maxwell in his essay more than 150 years earlier, although the cause is different. Maxwell has been such an important contributor to the study of Saturn that the ‘Maxwell Gap’ in the C ring was named in his honour. Finally, Murray quoted from Maxwell's essay about the rings: “We must either explain its motion on the principles of mechanics, or admit that, in the realms of Saturn, there can be motion regulated by laws which we are unable to explain.” And, said Murray, this is still true today.

Q: Is it true that ice particles from the moon Enceladus are incorporated in the rings?
A: Enceladus is outside the main ring system. It does produce ice particles but they make little contribution to the main rings.

Q: What observation was Maxwell trying to explain?
A: He was trying to explain the stability of the rings, and this is something that we are still attempting to do today. Maxwell showed that the rings had to be composed of individual particles.

Q: Where did the particles of water ice come from?
A: From the break-up of a comet or a satellite. One theory is that the “rocky core” of a satellite was absorbed by the planet while the water ice remained in orbit.

6. Demonstration: Maxwell, Colour Vision and the Future of Digital Communication
Peter Reid – Planning & Communications Project Manager, University of Edinburgh
Professor Harald Haas – Chair of Mobile Communications, School of Engineering, University of Edinburgh

Did James Clerk Maxwell take the world's first colour photo, as is widely believed? According to Peter Reid, the answer is “no,” at least technically speaking. But Maxwell's influence on colour, light and data transmission continues to gather momentum, said Reid. And Professor Harald Haas underlined this when he carried out the world's first demonstration of an enhanced Light-Fidelity (LiFi) system which is capable of simultaneous transmission of three independent videos by a single beam of white illuminating light, which was created by combining three different wavelengths (red, blue and green), using the methods as demonstrated by Maxwell in 1861.

Serendipity has always played a big role in science, said Reid: sometimes “you get it right by doing the wrong thing.” And in the case of Maxwell's colour image, scientific detective work has proved that what had seemed impossible to do at the time was not in fact what Maxwell appeared to achieve.

Maxwell's theories of colour are based on the evolutionary structure of the human eye, which has receptors for red, green and blue light. The sensitivities of these receptors overlap, however, and it is possible to fool the brain by using the appropriate combinations of red, green and blue photons. Reid demonstrated this by showing the audience three coloured circles of light (red, green and blue) and overlapping them until the light seemed white, even though everyone's eyes were actually ‘seeing’ red, green and blue photons: the three receptors are activated, and the brain perceives white. The fact that our eyes work in this way is what makes colour imaging possible.

Maxwell and the photographer Thomas Sutton made three images of a coloured tartan ribbon on separate black-and-white plates using different chemical filters – ferric thiocyanate for red, copper chloride for green and copper sulphate for blue. Then, at the Royal Institution in May 1861, Maxwell reconstructed the picture of the ribbon by projecting each image onto a wall using a Magic Lantern through the same chemical filters, a process that Reid repeated during his talk.

As Reid described, however, it was soon realised that the experiment should not have worked: emulsions of that period were entirely insensitive to red light… yet a red image *was* produced. After investigations during the ribbon centenary in the 1960s, it was discovered that red dyes of the 19^{th} Century reflected not just red light, but ultraviolet too and, by equal good fortune, the chemical chosen for the red filter was also transparent at those wavelengths; so the images Sutton recorded were of green, blue and ultra-violet.

This happy accident was not “the world's first colour photograph” – indeed, Maxwell was not demonstrating it as such. Its true legacy lies in its description of the way that the human eye perceives the world. Reid demonstrated this by showing a photograph displayed on a computer screen, and progressively zooming into the image until its underlying structure was visible. At the lowest level, all displays are made up of individual picture elements – pixels – and every single pixel is divided into red, green and blue components, with the intensity of each component varying to create the illusion of colour – thus “fooling” the eye. That is why “all digital devices owe their existence to Maxwell's legacy,” Reid concluded.

Haas then took over and began his demonstration by discussing Maxwell's legacy, and how his theories of colour and electromagnetism “will transform the future of mobile digital communications.”

Smartphones are now everywhere, said Haas – there are more of them than humans. They are also a utility, and like any other utility, there are problems of supply and demand. By the year 2018, 1.8 million years of high-definition video will be streamed over the wireless Internet every month. By the year 2020, the “Internet of things” will see an estimated 26 billion connected digital devices, including nine million phones. So how will we cope with all this traffic? We used to think the radio spectrum (3KHz to 300GHz) had plenty of space, but it's already being used more than expected. We face a major problem with bandwidth, but Maxwell's pioneering work on colour theory applied to communication theory come to the rescue – the visible light part of the electromagnetic spectrum is 1,000 times bigger than the radio spectrum. “It's like discovering a new 'oil well' for data communications,” said Haas.

Light emitting diodes (LEDs) can already be used for the transmission of large amounts of data. We transmit the data by changing the intensity of the light at frequencies at speeds of tens and hundreds of megaHertz. These change are much faster than our eyes can perceive, and up to three Gbps from a single colour LED light have been demonstrated. So what if white light is produced by multiple coloured LEDs or lasers? Haas’s research has shown that 100 Gbps wireless transmission speeds can be achieved with 36 wavelengths mixed to produce white light.

From Maxwell’s work on colour theory, we know that we can generate any colour from red, green and blue light. Can we use this principle of ‘orthogonality’ to transmit wirelessly three independent data streams from a single beam of white light, which at the same time illuminates the space in a very energy efficient manner? The challenge is to encode the data using different RGB lasers, combining the different colour beams into a single beam of white light, diffusing the light and extracting each data channel at randomly placed receivers within the cone of white light. “This is the modern version of Maxwell's light box turned into a high speed communication and illumination device,” said Haas, “and soon there will be one inside every light bulb.”

Haas then demonstrated the system – a world's first – and concluded by saying that this was the future of data transmission and illumination, thanks to Maxwell's theories of colour and light.

7. The Impact on Engineering of Maxwell's Articles on Structural Mechanics
Professor Iain MacLeod – Former Professor of Structural Engineering, University of Strathclyde

"Why did Maxwell take time out from his deep researches in electromagnetism and gas dynamics to solve problems in structural mechanics?" asked Professor MacLeod. The simple answer was that Maxwell was a practical man as well as a great theoretical scientist, who recognised the need for more reliable structures to reduce failures resulting in deaths. When a doctor makes an error, it may cause one death, said MacLeod, but if a structural engineer makes a mistake, it could result in thousands of deaths.

Structural mechanics is the mathematical logic and methods used to seek to ensure that structures perform satisfactorily. It may also be "the most used sector of applied science," MacLeod said, and Maxwell made a huge contribution which continues to influence the science today.

By spending time in structural mechanics, Maxwell followed some of the world's greatest thinkers, including Galileo, who tried to work out how to calculate the strength of a beam – a problem that would occupy the minds of some of the greatest scientists of Europe for the next 200 years. "The dawn of structural mechanics" was in 1826, when Louis Navier published his *Leçons,* which included much useful information in structural mechanics, including a solution to Galileo's bending problem.

Maxwell's first contribution, when he was not yet 19 years old, was his paper *On the equilibrium of elastic solids*, presented to the RSE in 1850. In this, he developed a novel approach to the derivation of the differential equations for an elastic solid and used these equations to provide solutions for a number of cases, some of which are still in use today.

MacLeod then described how Maxwell developed a method to calculate the internal forces in statically determinate triangulated frames (solutions for which only require the use of the principle of equilibrium), such as were commonly used for bridges. But his greatest contribution, Macleod said, was to develop a method for calculating the internal forces and stiffness of frames which are statically indeterminate, i.e., which have more members than necessary to make them stable. The calculation of such frames requires consideration of stiffness and compatibility (the way that the parts of the frame fit together) as well as equilibrium. To do this, he devised a number of techniques which became standard in the solution of frames. One of these, the principle of contragredience, is used extensively today in methods for computer solutions. Macleod described how, as a young researcher writing software for finite element solutions, he enjoyed using the principle because of its elegance.

The "solution method" for indeterminate frames that Maxwell devised is now known mainly as the 'flexibility method' but is also known as the 'Maxwell-Mohr' method, and a 'Clerk Maxwell reciprocal theorem,' also stated in the paper, was widely used.

Up to the computer era, Maxwell's method was the main technique for solving indeterminate frames, but its use was restricted by the effort needed to solve the basic simultaneous equations. For example, in the analysis of frames for airships in the 1920s, it could take four weeks of work for two people just to achieve one solution using Maxwell's method.

Maxwell's genius for representing physical behaviour in mathematical terms is exemplified by his postulation – in a letter to William Thompson (Kelvin) in 1854 – that plastic failure in a material would occur when the distortional strain energy reached a critical value. What is normally known as the 'von Mises criterion', based on this principle, is still in common use.

MacLeod’s conclusion was that “Maxwell turned the searchlight of his genius on structural mechanics and made a major contribution to it.”

8. The Impact on Control Theory and Cybernetics of Maxwell’s Paper on Governors

Professor Rodolphe Sepulchre – Professor of Engineering, University of Cambridge

We may think “cruise control” is something very new, or imagine that the “feedback mechanisms” which biologists study when developing drugs are a very modern idea, but James Clerk Maxwell paved the way for understanding many of these systems and is even regarded as the “father” of a new science called cybernetics or “the science of communication and control theory,” still in its infancy in 2015.

Maxwell's paper *On governors*, published in 1868, was “the first significant mathematical treatment of feedback mechanisms,” said Professor Sepulchre, and this is what provides the link with cruise control for cars. In the 1860s, there may not have been any cars, but different kinds of engines required speed regulation and used the feedback principle of correcting the engine torque based on the speed error. They all faced the the same basic problem of adjusting the feedback gain of the regulating device: the regulation was inefficient when the gain was too low and the engine could become unstable when the gain was too high. Nowadays, one line of code in some software determines the feedback gain, but in the early 19th Century, the feedback correction was the result of complicated mechanical links, and designing such *governors* was more an art than a science.

Maxwell first got interested in governors in 1861, at a time when he needed a speed regulation device for a physics experiment. Seven years later, he published his paper *On governors*. This work made several key contributions, said Sepulchre, proposing to analyse the stability question through a linearised model and establishing an important distinction between proportional and integral control.

Maxwell turned the stability question into the question of checking that the roots of a polynomial all have a negative real part. Even though he did not fully develop the solution of this problem in his work, his ideas inspired Edward John Routh, who won the Adams Prize in 1877 for his paper on “dynamic stability” – the first to cite Maxwell's earlier paper. Regarding the distinction between proportional and integral control, Maxwell noted that only integral control which makes the corrective action “proportional to the *integral* of the velocity error” could possibly achieve exact regulation of the speed, without residual error.

So why is Maxwell considered the father of control theory? Sepulchre suggested that his mathematical work on stability analysis and integral control were his main contribution, and they are still the first thing taught to students. Integral control eventually led to the internal model principle, a foundation of modern control theory. The ‘governor’ problem of the 19th Century later became the ‘repeater’ problem of the 20th Century, leading to the theory of the feedback amplifier. And in the 21st Century, integral control is recognised as an important part of regulation in biology, helping to understand how cells can tightly regulate various signalling mechanisms.

“Maxwell's time saw the detachment of pure science and applied science,” Sepulchre continued, but Maxwell did not think in terms of a “hierarchy of nobility” when it came to science versus engineering. “He foresaw that pure science is the preferred method of engineering, and that scientific principles should govern the design of artificial devices,” he added.

Finally, said Sepulchre, Maxwell's work provides the foundation of feedback science, which is the mathematical treatment of control theory and cybernetics.

Q: What is the relationship between machine learning and feedback mechanisms?
A: The feedback mechanism is designed to compensate for uncertainty, so it's different to machine learning, but there is a connection – after all, human beings learn by building an internal model of the environment.

9. The Impact of Maxwell's Demon on Information Theory and Computing
Professor Jim Al-Khalili FRAS HonFBAASc – Professor of Physics, University of Surrey

Professor Al-Khalili began by declaring that he would "resurrect the Demon," at least temporarily. Maxwell's Demon, he said, is all about the link between the physical world and the world of information. To explain the significance of the Demon, he then asked, "What is the most profound concept in physics?" Is it conservation, symmetry, the Big Bang or the composition of atoms? Many people believe that it is the Second Law of Thermodynamics – the idea that in every natural thermodynamic process, over time, entropy (disorder or a measure of the amount of energy which is unavailable to do work) increases but never decreases. The law is so central to physics that Einstein believed it was "the only theory that will never be overthrown."

In 1867, however, James Clerk Maxwell did a thought experiment to prove that it could "hypothetically be violated," and that the Second Law of Thermodynamics was only correct statistically. In the "experiment," he imagines two boxes placed side by side, one with hot gas inside it and the other with cold gas, like a fridge and an oven. If you allow the gas to move from one box to the other, eventually the particles will mingle and produce disorder or entropy. As Al-Khalili later explained, "low entropy" is analogous to a pack of cards arranged in perfect order, and high entropy or disorder is when you shuffle the pack. Thus, when the gases mix together at random, it's like shuffling the cards, and you would get the same result using pressure rather than gas.

Maxwell's "mischievous" idea was the paradox that simply by knowing what is going on inside the box, you can lower its entropy (increase the order). And to describe this hypothetical situation, he imagined a Demon observing the particles inside the box. Every time he sees a faster (hotter) particle, he opens a door in the partition separating the two sides of the box and allows the particle to join the other hot particles, and vice versa. Eventually, all the hot particles are on one side and all the cold particles on the other – a result which can only be achieved by having information. And this violates the Second Law of Thermodynamics.

According to Al-Khalili, this doesn't explain the whole situation, however, and Maxwell also raised concerns about it – kicking off more than a century of heated debate about whether the Demon "violated" or disproved the theory. Ludwig Boltzmann, the "father of statistical mechanics," was one of those who joined in the discussion through the years, and others followed, including Leo Szilard and Rolf Landauer.

Some physicists have suggested that you couldn't replace the Demon with a mechanical device, but all agree there is a link between entropy and information. Szilard – the inventor of the linear accelerator, the electron microscope and the cyclotron (for none of which he received due credit) – wrote a paper in 1929 entitled, *Reduction of entropy in a thermodynamic system by the interference of an intellgent being*. This, said Al-Khalili, was the first link between thermodynamics and information theory. Szilard suggested that the reduction of entropy in the boxes was more than compensated by the increase of entropy caused in the process of acquiring the information. Before gaining the data, the brain of the Demon is blank, and as he gains the data, its entropy increases. The expenditure of energy by the Demon will therefore cause an increase in the entropy in the brain of the Demon, which will be larger than the lowering of the entropy of the gas.

Landauer also thought we should consider the total entropy of the gas and the Demon combined, and suggested that as the Demon's brain filled up with information, he would have to erase information, and that this would increase entropy by dissipating heat.

In addition, opening and closing the door at random does not help, and measuring the energy also increases the entropy. Or perhaps, said Al-Khalili, we simply need a better definition of entropy.

To explain the subtleties of entropy further, Al-Khalili showed that if you have five playing cards of one suit in order (e.g. 2,3,4,5,6 of hearts), that represents low entropy, while a random order of cards (e.g. 2,5,4,3,6) is high entropy. Even though there is an equal chance of getting the same arrangement of cards, you need more information to describe the high entropy – i.e., it's easier to describe low entropy by saying “arrange cards in ascending order.”

“So, is the Demon exorcised?” asked Al-Khalili, suggesting it is still an open question. Some physicists even believe we need quantum theory to resolve it.

Finally, Al-Khalili wondered what Maxwell would have made of this continuing debate. And he concluded that the man who started it all in the first place “would have been on the side of the Demon,” to highlight the statistical nature of thermodynamics. On a more personal level, Al-Khalili also confessed that even though he can understand Maxwell's equations and how he worked out that light is an electromagnetic wave, Maxwell's Demon still gives him a headache.

Q: Can you relate this to cryptography?
A: There is a relationship between Maxwell's Demon, quantum cryptography and entanglement, and a link between thermodynamics, information theory and quantum theory.

Q: Does life not lead to order – e.g., in biology or human intelligence?
A: The order in life comes from replication, but the problem is establishing the first step in the process.

Q: When I reformat my laptop, it gets hot. Perhaps Szilard and Landauer were both right?
A: This reminds me of the question, does my iPhone get heavier when it is filled up with music?

The Vote of Thanks for offered by David Forfar FFA FRSE, Chairman of the James Clerk Maxwell Foundation.